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A backflow effect in smectic C liquid crystals in a bookshelf geometry

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We consider the instability that is induced when a large (supercritical) magnetic field is applied to a sample of smectic C liquid crystal held between parallel plates in a bookshelf geometry, with strong anchoring at the cell walls. Using a recently proposed dynamic theory, we solve the full linearized equations (with transportation of material between the smectic layers allowed) to show that the backflow that is induced has components both parallel and perpendicular to the layers, in contrast with the assumptions of Carlsson *et al.* The corresponding growth rate of the instability is obtained in terms of the applied field strength and material parameters.

1. Introduction

The use of smectic C liquid crystals in the development of electro-optical devices [1] has been the motivation for an extensive study of the director dynamics in these materials for more than a decade. In the absence of a viable dynamical theory, investigations have generally assumed that there is no coupling between macroscopic flow and director reorientation in these layered materials. However a recent paper by Leslie *et al.* [2] proposes a fully dynamic continuum theory for such materials, utilising the simplifying assumptions that the layer thickness remains constant and that the tilt of the director with respect to the layer normal remains unchanged. Employing this theory Carlsson *et al.* [3] investigated the importance of backflow on the switching behaviour of surface stabilized ferroelectric liquid crystal cells, while Leslie and Blake [4] examined its effect upon orientational relaxation in smectic C liquid crystals. However in their analysis Carlsson *et al.* [3] imposed the additional constraint that the velocity field is everywhere perpendicular to the layer normal, which means that the transportation of material between layers cannot occur. Although this further assumption simplifies the analysis, the solution found by Carlsson *et al.* [3] does not satisfy the linear momentum equation in the theory. In fact the introduction of an extra condition renders the system overdetermined.

In this paper we re-examine the problem that was considered by Carlsson *et al.* [3] in which a relatively large magnetic field is suddenly applied across a sample of smectic C liquid crystal in equilibrium in a bookshelf alignment between two large, parallel, horizontal flat plates. After giving a brief outline of the continuum theory in § 2, we formulate the problem for the particular type of Fréedericksz transition under consideration in terms of a linear stability analysis in § 3. However we choose to follow Leslie and Blake [4] and permit transportation of material between the layers. Although this leads to a rather more complicated system of equations than in [3], it is shown in § 4 that a similar method of solution yields results that appear to be qualitatively the same as those in [3], but are, of course, quantitatively different. Furthermore the solution presented here (unlike the one in [3]) satisfies the full set of continuum equations proposed by Leslie *et al.* [2].

2. The continuum equations

In this section we present a brief summary of the phenomenological equations governing the elastic–hydrodynamic behaviour of smectic C liquid crystals proposed by Leslie *et al.* [2]. Assuming that the smectic material consists of uniform layers with a fixed tilt of the alignment with respect to the layer normal, the constrained continuum theory introduces two orthonormal vectors to describe the smectic layered configuration. One is the unit normal to the layer **a** and the other is a unit vector **c** that is parallel to the layers and describes the direction of the tilt of the molecular alignment. Thus **a** and **c** must satisfy the constraints

$$\mathbf{a}\cdot\mathbf{a}=1,\quad \mathbf{c}\cdot\mathbf{c}=1,\quad \mathbf{a}\cdot\mathbf{c}=0, \tag{1}$$

while the further assumption that the medium is free of

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defects imposes the additional constraint

$$\operatorname{curl}\mathbf{a} = 0. \tag{2}$$

With the assumption of incompressibility, the additional equations required to determine $\mathbf{a}, \mathbf{c}$ and the velocity vector field $\mathbf{v}$ are, in Cartesian tensor notation, the constraint

$$v_{i,i} = 0, \tag{3}$$

the linear momentum equation

$$\rho \dot{v}_i = -\tilde{p}_{,i} + \tilde{g}^a_j a_{j,i} + \tilde{g}^c_j c_{j,i} + \tilde{t}_{ij,j}, \tag{4}$$

and the angular momentum equations

$$\left(\frac{\partial W}{\partial a_{i,j}}\right)_{,j} - \frac{\partial W}{\partial a_i} + \tilde{g}^a_i + G^a_i + \varepsilon_{ijk}\beta_{k,j} + \gamma a_i + \kappa c_i = 0 \tag{5}$$

and

$$\left(\frac{\partial W}{\partial c_{i,j}}\right)_{,j} - \frac{\partial W}{\partial c_i} + \tilde{g}^c_i + G^c_i + \kappa a_i + \tau c_i = 0, \tag{6}$$

where

$$\left.\begin{aligned}
\tilde{p} &= -H_{\mathrm{m}} + p + W, \quad \tilde{t}_{ij} = \tilde{t}^{s}_{ij} + \tilde{t}^{ss}_{ij},\\
\tilde{t}^{s}_{ij} &= \mu_0 D_{ij} + \mu_1 a_p D^a_p a_i a_j + \mu_2(D^a_i a_j + D^a_j a_i)\\
&\quad + \mu_3 c_p D^c_p c_i c_j + \mu_4(D^c_i c_j + D^c_j c_i)\\
&\quad + \mu_5 c_p D^a_p(a_i c_j + a_j c_i) + \lambda_1(A_i a_j + A_j a_i)\\
&\quad + \lambda_2(C_i c_j + C_j c_i) + \lambda_3 c_p A_p(a_i c_j + a_j c_i)\\
&\quad + \kappa_1(D^a_i c_j + D^a_j c_i + D^c_i a_j + D^c_j a_i)\\
&\quad + \kappa_2\{a_p D^a_p(a_i c_j + a_j c_i) + 2a_p D^c_p a_i a_j\}\\
&\quad + \kappa_3\{c_p D^c_p(a_i c_j + a_j c_i) + 2a_p D^c_p c_i c_j\}\\
&\quad + \tau_1(C_i a_j + C_j a_i) + \tau_2(A_i c_j + A_j c_i)\\
&\quad + 2\tau_3 c_p A_p a_i a_j + 2\tau_4 c_p A_p c_i c_j,\\
\tilde{t}^{ss}_{ij} &= \lambda_1(D^a_j a_i - D^a_i a_j) + \lambda_2(D^c_j c_i - D^c_i c_j)\\
&\quad + \lambda_3 c_p D^a_p(a_i c_j - a_j c_i) + \lambda_4(A_j a_i - A_i a_j)\\
&\quad + \lambda_5(C_j c_i - C_i c_j) + \lambda_6 c_p A_p(a_i c_j - a_j c_i)\\
&\quad + \tau_1(D^a_j c_i - D^a_i c_j) + \tau_2(D^c_j a_i - D^c_i a_j)\\
&\quad + \tau_3 a_p D^a_p(a_i c_j - a_j c_i) + \tau_4 c_p D^c_p(a_i c_j - a_j c_i)\\
&\quad + \tau_5(A_j c_i - A_i c_j + C_j a_i - C_i a_j),\\
\tilde{g}^a_i &= -2(\lambda_1 D^a_i + \lambda_3 c_p D^a_p c_i + \lambda_4 A_i + \lambda_6 c_p A_p c_i\\
&\quad + \tau_2 D^c_i + \tau_3 a_p D^a_p c_i + \tau_4 c_p D^c_p c_i + \tau_5 C_i),\\
\tilde{g}^c_i &= -2(\lambda_2 D^c_i + \lambda_5 C_i + \tau_1 D^a_i + \tau_5 A_i),\\
D^a_i &= D_{ij} a_j, \quad D^c_i = D_{ij} c_j, \quad 2D_{ij} = v_{i,j} + v_{j,i},\\
A_i &= \dot{a}_i - W_{ij} a_j, \quad C_i = \dot{c}_i - W_{ij} c_j,\\
2W_{ij} &= v_{i,j} - v_{j,i}.
\end{aligned}\right\} \tag{7}$$

Here ρ is the constant density, ε_{ijk} is the alternator and a superposed dot indicates a material time derivative. The arbitrary scalar functions p, γ, τ, κ and arbitrary vector function β arise from the constraints (3), (1) and (2), respectively, while G^a and G^c denote any generalized external body forces acting and H_{m} represents the energy per unit volume due to the presence of any electric or magnetic field. Of particular importance to this paper are those forces associated with an applied magnetic field $\mathbf{H}$ which take the form

$$G^a = \chi_{\mathrm{a}}(\mathbf{H}\cdot\mathbf{n})\mathbf{H}\cos\alpha, \quad G^c = \chi_{\mathrm{a}}(\mathbf{H}\cdot\mathbf{n})\mathbf{H}\sin\alpha, \tag{8}$$

where χ_{a} denotes the anisotropic part of the magnetic susceptibility (assumed constant), and α is the fixed tilt angle between the layer normal $\mathbf{a}$ and the average molecular alignment $\mathbf{n}$, with $\mathbf{n} = \mathbf{a}\cos\alpha + \mathbf{c}\sin\alpha$. Also W is the elastic stored-energy per unit volume, taking the form [4]

$$\begin{aligned}
2W &= K^a_1(a_{i,i})^2 + K^c_1(c_{i,i})^2 + K^a_2(c_i a_{i,j} c_j)^2\\
&\quad + K^c_2 c_{i,j} c_{i,j} + K^c_3 c_{i,j} c_j c_{i,k} c_k\\
&\quad + 2K^a_3 a_{i,i}(c_j a_{j,k} c_k) + 2K^c_4 c_{i,j} c_j c_{i,k} a_k\\
&\quad + 2K^{ac}_1 c_{i,i}(c_j a_{j,k} c_k) + 2K^{ac}_2 a_{i,i} c_{j,j}.
\end{aligned} \tag{9}$$

The theory thus provides 16 equations (1)–(6) to determine the sixteen variables a_i, c_i, v_i, β_i, p, γ, κ and τ. If one follows Carlsson *et al.* [3] in introducing the extra equation $\mathbf{v}\cdot\mathbf{a} = 0$ into the theory, the system becomes overdetermined and it is generally not possible to find solutions that satisfy all the equations, even for the simplest of experimental arrangements (as illustrated by the analyses of Carlsson *et al.* [3] and Leslie and Blake [4]). For this reason we do not adopt this additional constraint, and hence we allow the transportation of material between layers.

3. Formulation of the problem

Suppose a sample of smectic liquid crystal is confined between two large, horizontal flat plates with the uniform layers perpendicular to the bounding planes. We assume that the direction of the tilt angle is uniform throughout the sample, and consider the application of a uniform magnetic field $\mathbf{H}$ applied perpendicular to the initial uniform molecular alignment. Cartesian coordinate axes are chosen so that the upper and lower plates occupy the planes $z = d/2$ and $z = -d/2$, respectively, the normal $\mathbf{a}$ to the layers is in the x-direction, and the applied magnetic field is given by

$$\mathbf{H} = (0, 0, H), \tag{10}$$

where H is a constant.

The uniformly aligned equilibrium configuration

$$\mathbf{v} = 0, \quad \mathbf{a} = (1, 0, 0), \quad \mathbf{c} = (0, 1, 0) \tag{11}$$

(with $\tilde{p}$ being a constant p_0) is one obvious solution of the equations (1)–(6), and it is well known that this configuration obtains until H exceeds a critical value H_c given by $H_c = ((K_1^c + K_2^c)/\chi_a)^{1/2}(\pi/d \sin\alpha)$, when a Fréedericksz transition occurs. Here we are interested in the initial dynamics associated with this instability, in particular the effect of backflow when a magnetic field with strength $H > H_c$ is suddenly applied across the sample. Since we are concerned only with the dynamics at the start of the instability, we seek solutions of the form

$$\mathbf{v} = (\hat{u}(z,t), \hat{v}(z,t), 0), \quad \mathbf{a} = (1,0,0), \quad \mathbf{c} = (0,1,\hat{\phi}(z,t)), \tag{12}$$

with $\tilde{p} = p_0 + \hat{p}$, where $\hat{u}$, $\hat{v}$, $\hat{\phi}$ and $\hat{p}$ and their derivatives are small compared to unity. One observes that, as is customary, the layer normal is assumed to remain fixed in its initial direction. With (12), the constraints (1)–(3) are satisfied identically, and a linearization of equations (6) and (4) results in the equations

$$(K_1^c + K_2^c)\hat{\phi}_{zz} - (\lambda_2 + \lambda_5)\hat{v}_z - (\tau_1 + \tau_5)u_z - 2\lambda_5\hat{\phi}_t + (\chi_a \mathrm{H}^2 \sin^2\alpha)\phi = 0, \tag{13}$$

$$(\mu_0 + \mu_2 + 2\lambda_1 + \lambda_4)u_{zz} + (\kappa_1 + \tau_1 + \tau_2 + \tau_5)v_{zz} + 2(\tau_1 + \tau_5)\hat{\phi}_{zt} = 0, \tag{14}$$

$$(\mu_0 + \mu_4 + 2\lambda_2 + \lambda_5)\hat{v}_{zz} + (\kappa_1 + \tau_1 + \tau_2 + \tau_5)\hat{u}_{zz} + 2(\lambda_2 + \lambda_5)\hat{\phi}_{zt} = 0, \tag{15}$$

together with

$$\kappa = \tau = 0, \quad \hat{p} = \text{constant}. \tag{16}$$

Introducing non-dimensional variables $\tilde{z}$, $\tilde{t}$, U and V defined by

$$z = \tilde{z}d, \quad t = \frac{\tilde{t}\lambda_5}{\chi_a H_c^2 \sin^2\alpha}, \quad \hat{u} = U\frac{d\chi_a H_c^2 \sin^2\alpha}{\lambda_5}, \quad \hat{v} = V\frac{d\chi_a H_c^2 \sin^2\alpha}{\lambda_5} \tag{17}$$

(well-defined since $\lambda_5 > 0$) and seeking solutions of the form

$$\hat{\phi} = \tilde{\phi}(\tilde{z})e^{s\tilde{t}}, \quad U = \tilde{u}(\tilde{z})e^{s\tilde{t}}, \quad V = \tilde{v}(\tilde{z})e^{s\tilde{t}}, \tag{18}$$

where s is a constant dimensionless growth rate, we find that the system of equations (13)–(15) becomes

$$(D^2 - A_1)\tilde{\phi} - \pi^2 A_2 D\tilde{u} - \pi^2 A_3 D\tilde{v} = 0, \tag{19}$$

$$D^2\tilde{u} + A_4 D^2\tilde{v} + 2A_5 sD\tilde{\phi} = 0, \tag{20}$$

$$A_6 D^2\tilde{u} + D^2\tilde{v} + 2A_7 sD\tilde{\phi} = 0, \tag{21}$$

where

$$\left.\begin{aligned} A_1 &= (2s - h^2)\pi^2, \\ A_2 &= (\tau_1 + \tau_5)/\lambda_5, \\ A_3 &= (\lambda_2 + \lambda_5)/\lambda_5, \\ A_4 &= \frac{\kappa_1 + \tau_1 + \tau_2 + \tau_5}{\mu_0 + \mu_2 + 2\lambda_1 + \lambda_4}, \\ A_5 &= \frac{\tau_1 + \tau_5}{\mu_0 + \mu_2 + 2\lambda_1 + \lambda_4}, \\ A_6 &= \frac{\kappa_1 + \tau_1 + \tau_2 + \tau_5}{\mu_0 + \mu_4 + 2\lambda_2 + \lambda_5}, \\ A_7 &= \frac{\lambda_2 + \lambda_5}{\mu_0 + \mu_4 + 2\lambda_2 + \lambda_5}, \\ h^2 &= \frac{H^2}{H_c^2}, \end{aligned}\right\} \tag{22}$$

and $D \equiv \mathrm{d}/\mathrm{d}\tilde{z}$. For convenience the tildes will now be dropped.

We assume that at the bounding plates, the director fields satisfy a strong-anchoring condition and the velocity field satisfies a no-slip condition, so that

$$\phi\left(\pm\frac{1}{2}\right) = u\left(\pm\frac{1}{2}\right) = v\left(\pm\frac{1}{2}\right) = 0. \tag{23}$$

4. Solution

Extending a method employed by Brochard *et al.* [5] and subsequently Carlsson *et al.* [3], we seek solutions having the form

$$\begin{aligned} \phi &= \phi_0\left(\cos qz - \cos\left(\frac{1}{2}q\right)\right), \\ u &= u_0\left(\sin qz - 2z\sin\left(\frac{1}{2}q\right)\right), \\ v &= v_0\left(\sin qz - 2z\sin\left(\frac{1}{2}q\right)\right), \end{aligned} \tag{24}$$

with q non-zero. The boundary conditions (23) are automatically satisfied, and substitution of (24) into (19)–(21) yields the linear algebraic equations

$$\left\{(-q^2 - A_1)\cos qz + A_1\cos\left(\frac{1}{2}q\right)\right\}\phi_0 - \pi^2\left(q\cos qz - 2\sin\left(\frac{1}{2}q\right)\right)(A_2 u_0 + A_3 v_0) = 0, \tag{25}$$

$$q^2 u_0 + A_4 q^2 v_0 + 2A_5 sq\phi_0 = 0, \tag{26}$$

$$A_6 q^2 u_0 + q^2 v_0 + 2A_7 sq\phi_0 = 0. \tag{27}$$

Equating coefficients in (25) results in the equations

$$(q^2 + A_1)\phi_0 + \pi^2 q(A_2 u_0 + A_3 v_0) = 0, \tag{28}$$

$$A_1 \cos\left(\frac{1}{2}q\right)\phi_0 + 2\pi^2 \sin\left(\frac{1}{2}q\right)(A_2 u_0 + A_3 v_0) = 0. \tag{29}$$

Equations (26)–(29) constitute a linear algebraic system of the form

$$\boldsymbol{M}\Phi = 0, \quad \Phi = (\phi_0, u_0, v_0)^{\mathrm{T}}, \tag{30}$$

where $\boldsymbol{M}$ is a 4×3 matrix and the exponent T denotes a transpose. The conditions for this system to have a non-trivial solution Φ are simply

$$s = \frac{2}{\pi^2 A}\left[\frac{X^3}{X - \tan X}\right], \quad h^2 = \frac{4X^2}{\pi^2}\left[\frac{(X/A) - \tan X}{X - \tan X}\right], \tag{31 a,b}$$

where

$$X := \frac{1}{2}q,$$

$$A := \{A_2(A_5 - A_4 A_7) + A_3(A_7 - A_5 A_6)\}/(1 - A_4 A_6). \tag{32}$$

The formal solution can now be completed by taking the divergence of (5), solving the resulting differential equation for γ, and then using (5) to determine $\operatorname{curl}\beta$, if desired.

Equation (31) gives the 'wavenumber' X implicitly in terms of the applied field h, and also gives the growth rate s parametrically in terms of h (with parameter X). By elimination of X the h–s relation may alternatively be written in the implicit form

$$As \tan\left[\frac{1}{2}\pi(h^2 - 2s + 2As)^{1/2}\right] = -\frac{1}{4}\pi(h^2 - 2s + 2As)^{1/2}(h^2 - 2s). \tag{33}$$

Surprisingly the results in (31) have the same general structure as equations (55)–(56) of Carlsson *et al.* [3], but with a different expression for the parameter A in terms of viscosities. (Superficially the results look different; however their parameter β is not a constant: it depends on the strength of the applied field (our h). When this is substituted, the two results are seen to be of the same form.) It is worth noting that in the special case of a smectic C_M phase, the material parameters κ_i and τ_i in (7) are zero (for all i), so that in (22) we have $A_2 = A_4 = A_5 = A_6 = 0$. Then (19)–(21) lead to the reduced system

$$(D^2 - A_1)\tilde{\phi} - \pi^2 A_3 D\tilde{v} = 0, \quad D^2\tilde{u} = 0,$$
$$D^2\tilde{v} + 2A_7 s D\tilde{\phi} = 0, \tag{34}$$

whose solution (subject to (23)) is again (24), with $u_0 = 0$ and with s, h and X related by (31), but with A defined by $A := A_3 A_7$. System (34) is essentially equivalent to the one considered by Carlsson *et al.* [3] (though they assumed $u \equiv 0$ at the outset); thus in the case of a smectic C_M material, their no-permeation ($u \equiv 0$) assumption leads to a consistent solution, and our results tally exactly with theirs in that case.

5. Discussion

The h–s relation (31) or (33) has infinitely many branches, for any value of the material parameter A. It seems that this relation can make sense physically only if A satisfies

$$0 < A < 1. \tag{35}$$

For a given A inside this interval, the branch on which the growth rate s is largest (the 'lowest' branch) is the one that determines the stability of the system. It is found that instability arises (i.e. $s > 0$) for $h > 1$, with the lowest branch corresponding to $\frac{1}{2}\pi \leqslant X < X_0$, where X_0 ($\approx 4{\cdot}49$) is the smallest positive root of the equation $X_0 = \tan X_0$. 'Higher' branches of the h–s curve correspond either to stable modes or to unstable modes with a smaller value of s, at any given field strength h. For $A > 1$ the growth rate s is positive even for $h = 0$, showing that the initial bookshelf state would be 'mechanically unstable' even with no magnetic field present. Not only that, but in this case the higher branches correspond to modes with larger (positive) s, and indeed even at $h = 0$ there are modes with indefinitely large values of s, which presumably is unphysical. For $A < 0$ there are jump discontinuities in the h–s relation for $s > 0$, meaning that the material would exhibit discontinuous behaviour, which is not to be expected physically. In particular, for any $A < 0$ the system is unstable with a finite growth rate $s_J := 6/\pi^2|A|$ (corresponding to small X) for $h = h_J^-$, but is stable for $h = h_J^+$, where $h_J := [12(A-1)/\pi^2 A]^{1/2}$; presumably such a jump in behaviour is unphysical. These observations suggest that only the case (35) can be of interest in modelling a smectic material that can attain an initial bookshelf configuration.

Interestingly a 'conventional' static analysis predicts correctly the bifurcation point $h = 1$ (as asserted earlier), but the natural assumption from this, that the system is stable for $h < 1$ and unstable for $h > 1$, is valid only if $0 < A < 1$, as shown by our dynamical analysis. Furthermore the material parameter A depends only on viscosity coefficients, which do not enter the static considerations.

Given that the only difference between the forms of our solution and that of Carlsson *et al.* [3] is the expression for A in terms of viscosities (which themselves have not yet been measured experimentally!), we may assert that their discussion from page 471 onwards carries over to our solution, and so need not be repeated here. In particular they present figures showing, as functions of h, comparisons of the response time $1/s$ and the wavenumber q with the corresponding quantities obtained when backflow is ignored (though strictly there is no such solution of the governing equations). Their diagrams (for $A = 0{\cdot}01$, $0{\cdot}1$, $0{\cdot}5$ and $0{\cdot}9$) carry over to our solution; perhaps all that needs to be emphasized is that the response time is reduced when flow effects are taken into account. We may demonstrate this reduction in the response time as follows. First we set $u = v = 0$ in (19) and (23), and, following Carlsson *et al.* [3], ignore equations (20) and (21), to obtain

$$(D^2 - A_1)\phi = 0, \quad \phi\left(\pm\frac{1}{2}\right) = 0, \quad A_1 := (2s_0 - h^2)\pi^2, \tag{36}$$

where s_0 is the growth rate for the 'negligible-backflow' transition. This gives straight forwardly

$$s_0 = \frac{1}{2}(h^2 - 1). \tag{37}$$

For a given h (>1) we wish to compare s in (31 *a*) with s_0 here. By (31 *b*) and (37) we may write

$$s_0 = \frac{2X^2}{\pi^2}\left[\frac{(X/A) - \tan X}{X - \tan X}\right] - \frac{1}{2}, \tag{38}$$

and then we have

$$s - s_0 = \frac{1}{2} + \frac{2X^2 \tan X}{\pi^2(X - \tan X)} \quad \left(\frac{1}{2}\pi \leqslant X < X_0\right), \tag{39}$$

which, we note, is independent of A. It is easy to show that the right hand side here is positive over the interval $\frac{1}{2}\pi \leqslant X < X_0$ (for any A), so that $s > s_0$, meaning that the response time $1/s$ is less than the prediction $1/s_0$ that is obtained when backflow is neglected. This reduction in response time is found to be small when A is small, but becomes much more significant as A approaches unity ([see [3]).

Finally we remark that, following [5] and [3], we have considered only solutions of the type (24); there exist other solutions of (19)–(23), but these involve nonzero net fluxes parallel to the plates.

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Note added in proof

It has been suggested by a referee that there may exist solutions that involve nonzero pressure gradients parallel to the plates (unlike the solutions presented here and in [4]); this possibility is being investigated.